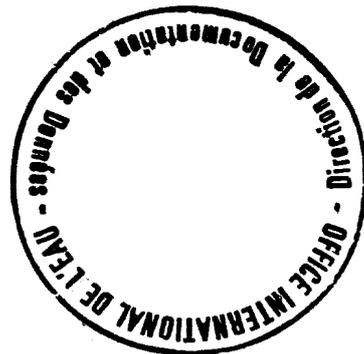


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STATISTICAL METHODS IN WATER RESOURCES

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