Sluice-Gate Discharge Equations

By Prabhata K. Swamee[1]

Abstract: Sluice-gate discharge coefficient is an involved function of geometric and hydraulic parameters. For free flow, it is related to upstream depth and gate opening, whereas for submerged flow, in addition to these parameters, it depends on tail-water depth. The present practice of discharge coefficient determination is from curves drawn relating discharge coefficient and upstream-depth gate-opening ratio with tail-water–depth gate-opening ratio being the third parameter for submerged flow. This method lends itself to sufficient error of judgment due to interpolation between two types of curves. Furthermore, the graphical information cannot be used for any analytical and/or numerical method for flow control and flow profile determination. In this paper high-accuracy discharge coefficient equations for free and submerged flows have been developed along with the criterion for determination of free and submerged flow. Procedures for solution of various gate-opening problems have also been given.

Introduction

A sluice gate is an opening in a hydraulic structure used for controlling the discharge. Fig. 1 shows flow through a sluice gate with no side or bottom contraction. Downstream free flow occurs at a (relatively) large ratio of upstream depth to the gate-opening height. However, submerged flow at the downstream would occur for low values of this ratio. For a freely issuing stream from a sluice gate, the water surface is quite smooth whereas for a submerged flow, the corresponding flow profile is extremely rough.

The conventional sluice-gate discharge equation is written in the form:

$$Q = C_d ab\sqrt{2gh_0} \qquad (1)$$

in which Q = the sluice-gate discharge; a = the sluice-gate opening; b = the sluice-gate length; h_0 = the upstream water depth; g = gravitational acceleration; and C_d = discharge coefficient. Fig. 2 shows the variation of C_d under free and submerged flow conditions as obtained by Henry (1950). Henry's experimental investigation is considered most extensive and reliable. Henry's investigation was later confirmed by Rajaratnam and Subramanya (1967).

Presented herein are accurate equations of discharge coefficient for free and submerged flow conditions. It is hoped that these equations will find use in flow regulation in canals and flow analysis involving sluice gates.

Discharge Coefficient Equations

Free Flow Condition

A perusal of Fig. 2 indicates for free flow the discharge coefficient progressively increases to a saturation value of 0.611. Hydraulically the sluice gate ceases to exist when $h_0 = a$ or less. Thus for $h_0 = a$ the discharge

[1]Prof. of Civ. Engrg., Univ. of Roorkee, Roorkee—247 667, India.

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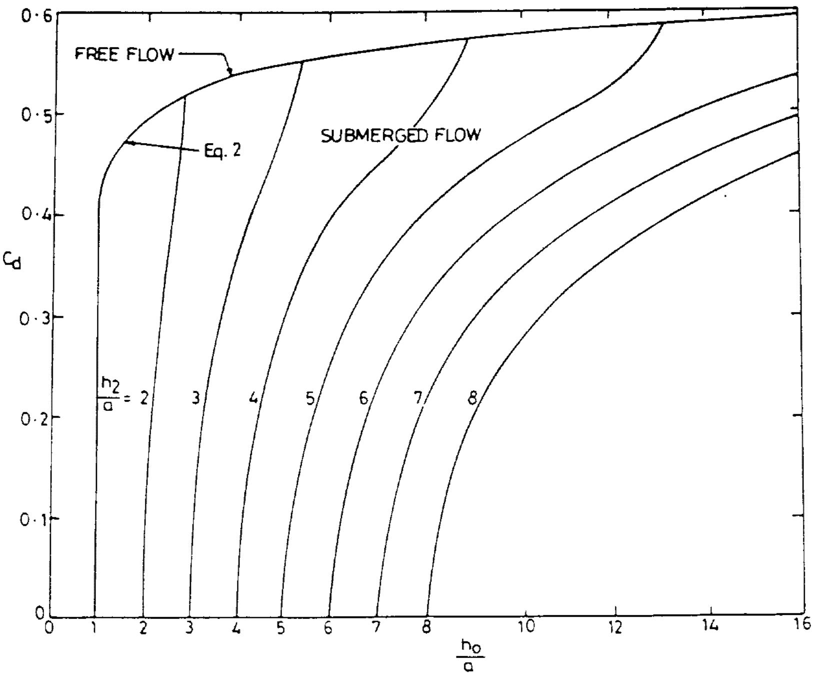


FIG. 2. Variation of Discharge Coefficient

at which the flow is free. From Fig. 2 h_{0max}/a can be obtained for various values of h_2/a. Plotting these values on a double logarithmic paper, the following equation is obtained:

$$\frac{h_{0max}}{a} = 0.81\left(\frac{h_2}{a}\right)^{1.72} \quad \ldots\ldots (4)$$

So long as h_0 lies between h_2 and h_{0max} submerged flow conditions will prevail. That is, for submerged flow to exist, the following condition is to be satisfied:

$$h_2 < h_0 < 0.81h_2\left(\frac{h_2}{a}\right)^{0.72} \quad \ldots\ldots (5a)$$

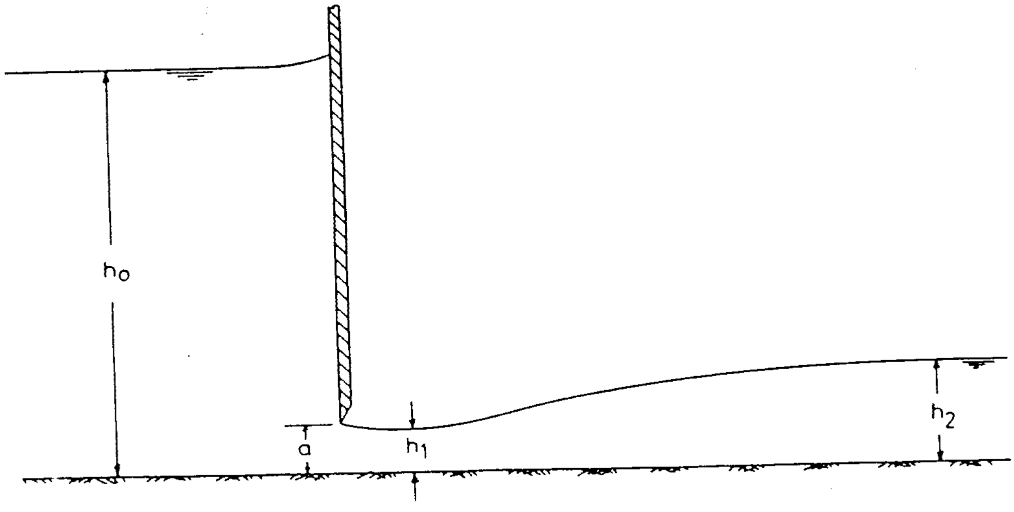
On the other hand, the condition for existence of free flow is:

$$h_0 \geq 0.81h_2\left(\frac{h_2}{a}\right)^{0.72} \quad \ldots\ldots (5b)$$

Using the discharge coefficient curves of Fig. 2 for submerged flow, the following equation has been fitted to describe the curves:

$$C_d = 0.611\left(\frac{h_0 - a}{h_0 + 15a}\right)^{0.072}(h_0 - h_2)^{0.7}\left\{0.32\left[0.81h_2\left(\frac{h_2}{a}\right)^{0.72} - h_0\right]^{0.7} + (h_0 - h_2)^{0.7}\right\}^{-1} \quad \ldots\ldots (6)$$

Eq. (6) is valid for the conditions described in 5(a). It can be seen that for $h_0 = h_{0max}$, (6) reduces to (2), which is the upper limit of its applicability. Similarly at the lower limit of its applicability for $h_0 = h_2$, Eq. (6) yields $C_d = 0$. For intermediate values of h_0, the difference between the values



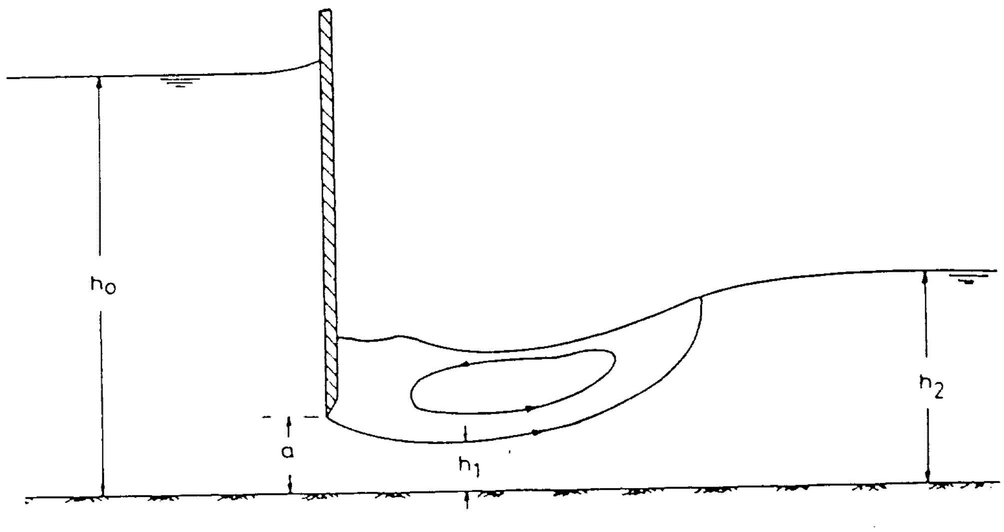


FIG. 1. Definition Sketch: (*a*) Free Flow; and (*b*) Submerged Flow

coefficient $C_d = 0$. Considering this limiting value and the coordinates of the free flow discharge curve, the following equation is obtained:

$$C_d = 0.611 \left(\frac{h_0 - a}{h_0 + 15a}\right)^{0.072} \quad \text{(2)}$$

Eq. (2) has an excellent agreement with the experimental curve of Fig. 2. Using (1) and (2) the following equation for the sluice-gate discharge is obtained:

$$Q = 0.864\, ab\sqrt{g h_0} \left(\frac{h_0 - a}{h_0 + 15a}\right)^{0.072} \quad \text{(3)}$$

Submerged Flow Condition

Under submerged flow conditions the discharge coefficient is zero when $h_0 = h_2$ (the tailwater depth). Any increase in h_0 above h_2 results in rapid increase in the discharge coefficient until h_0 attains a maximum value $h_{0\max}$

3. Algorithms for solution of practical problems of the sluice-gate operation have been given.

Appendix I. References

Henry, H. R. (1950). Discussion of "Diffusion of Submerged Jets," by M. L. Albertson, Y. B. Dai, R. A. Jensen, and H. Rouse. *Trans.*, ASCE, 115, 687–694.

Rajaratnam, N., and Subramanya, K. (1967). "Flow equations for the sluice gate." *J. of Irrig. and Drain. Engrg.*, ASCE, 93(3), 167–186.

Appendix II. Notations

The following symbols are used in this paper:

a = sluice-gate opening;
b = sluice-gate length;
C_d = discharge coefficient;
g = gravitational acceleration;
h_0 = upstream depth;
h_{0max} = maximum value of h_0 for submerged flow;
h_1 = depth at vena contracta;
h_2 = tail-water depth; and
Q = discharge.

obtained by (6) and that from Fig. 2 is negligible. Combining (1) and (6), the sluice-gate discharge equation is given by:

$$Q = 0.864\, ab\sqrt{gh_0}\left(\frac{h_0 - a}{h_0 + 15a}\right)^{0.072} (h_0 - h_2)^{0.7}$$

$$\cdot\left\{0.32\left[0.81h_2\left(\frac{h_2}{a}\right)^{0.72} - h_0\right]^{0.7} + (h_0 - h_2)^{0.7}\right\}^{-1} \quad \ldots\ldots\ldots\ldots\ldots (7)$$

Practical Applications

The methodology developed in the preceding sections can be used to solve the following commonly occurring problems in the sluice-gate operation:

1. To determine whether the flow is free or submerged [This problem can be solved by application of 5(a) and 5(b)].
2. To predict the discharge [The discharge can be predicted by application of (3) for free flow; and (7) for submerged flow as per flow conditions determined earlier].
3. To decide the gate opening for a given discharge [In this case flow can be either free or submerged. Take an arbitrary gate opening and using 5(a) and 5(b) to find the flow condition and accordingly calculate the discharge using either (3) (for free flow) or (7) (for submerged flow). If the calculated discharge is greater than the given discharge, reduce the gate opening, otherwise increase it, and repeat the process until the difference between the calculated and the given discharge is small].
4. To decide the upstream depth for a given discharge [Take a trial value of h_0 and find the flow condition using 5(a) and 5(b) to accordingly obtain the discharge using (3) or (7). If the calculated discharge is more than the given discharge, reduce the upstream depth, otherwise increase it and repeat the process, until the difference between the calculated and given discharge is small].
5. To decide the tail-water depth for a given discharge [The tail-water depth has to be increased if the free flow discharge is larger than the given discharge. Take a trial value of h_2 ensuring the submerged flow condition described by (5a) and calculate the discharge using (7). If the calculated discharge is more than the given discharge, increase the tail-water depth, otherwise reduce it, such that the condition of 5(a) is followed, and repeat the process until the difference between the calculated and the given discharge is small].

These algorithms can be adopted on computer-controlled sluice-gate operations in an irrigation canal network.

Conclusions

From the foregoing developments, the following conclusions are drawn:

1. Criteria for the existence of submerged and free flows through a sluice gate have been evolved.
2. High-accuracy discharge coefficient equations for submerged and free flows have been obtained.